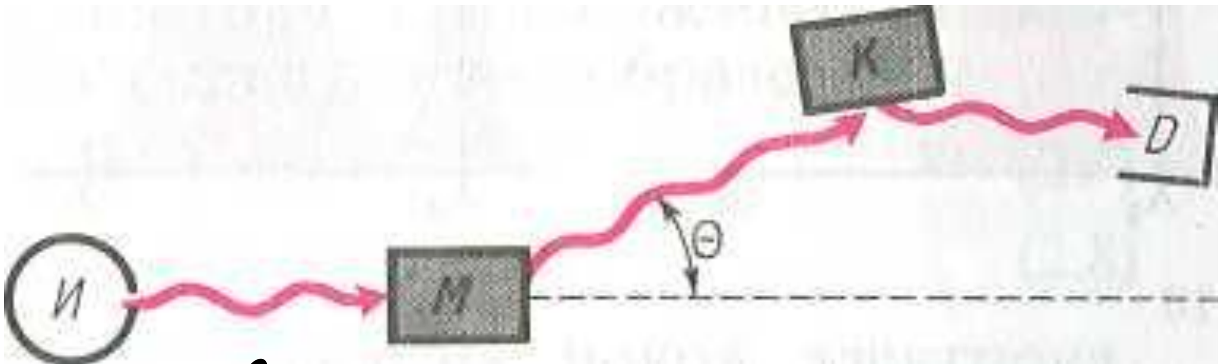
Lecture 3 . The Compton Effect. De Broglie Waves. The Theory of Wave-Particle Duality.

Glossary 3

wave -particle duality - корпускулярно-волновой дуализм
wave function - волновая функция
Incident photon - падающий фотон
scattered photon- рассеянный фотон
Plain wave –плоская волна
Probability – вероятность
Density of probability – плотность вероятности

The Compton effect



λ_0

Incident radiation

Scattered radiation λ_0 , λ

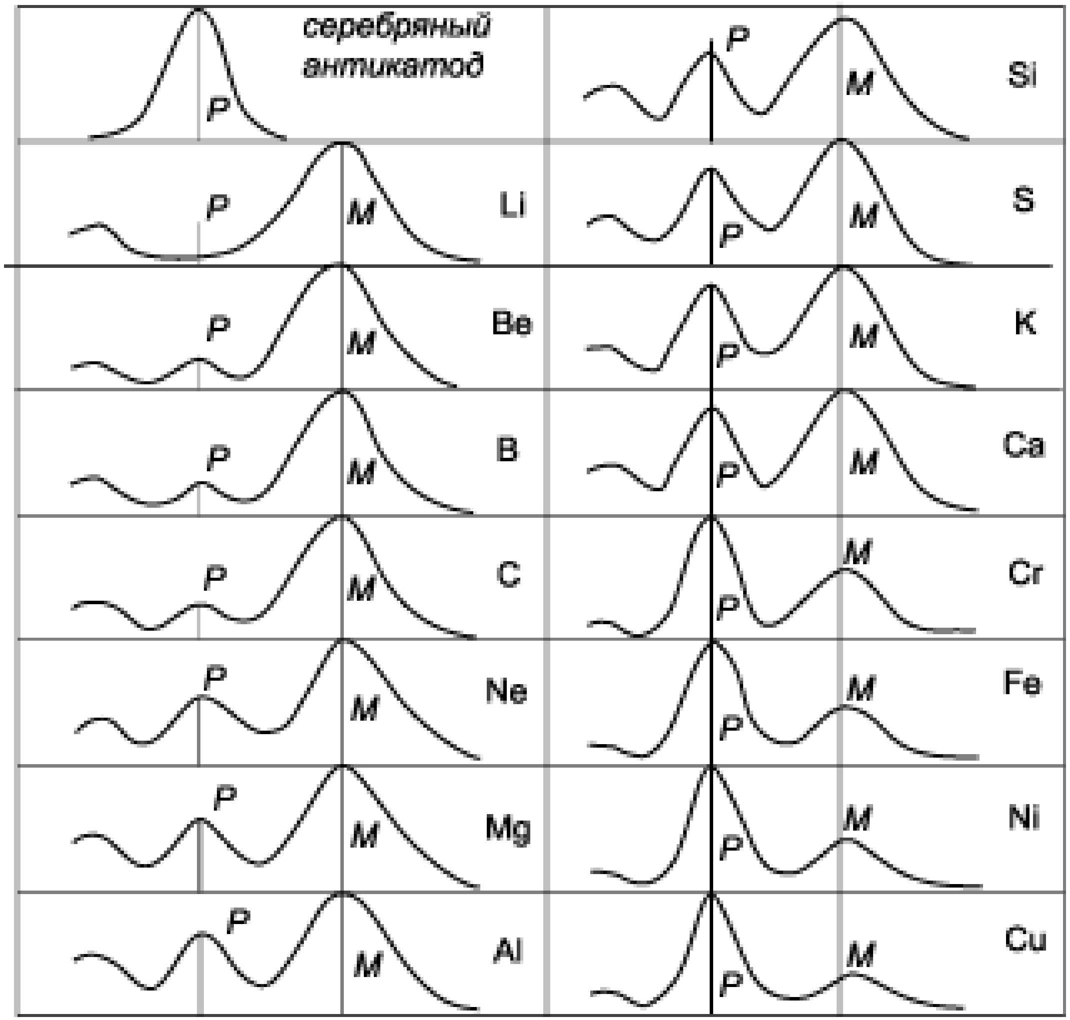


Рис. Зависимость спектра рассеиваемых фотонов от заряда ядра рассеивателя.

$$\Delta\lambda = 0.048 \cdot 10^{-10} \sin^2(\theta/2) \quad \text{м}$$

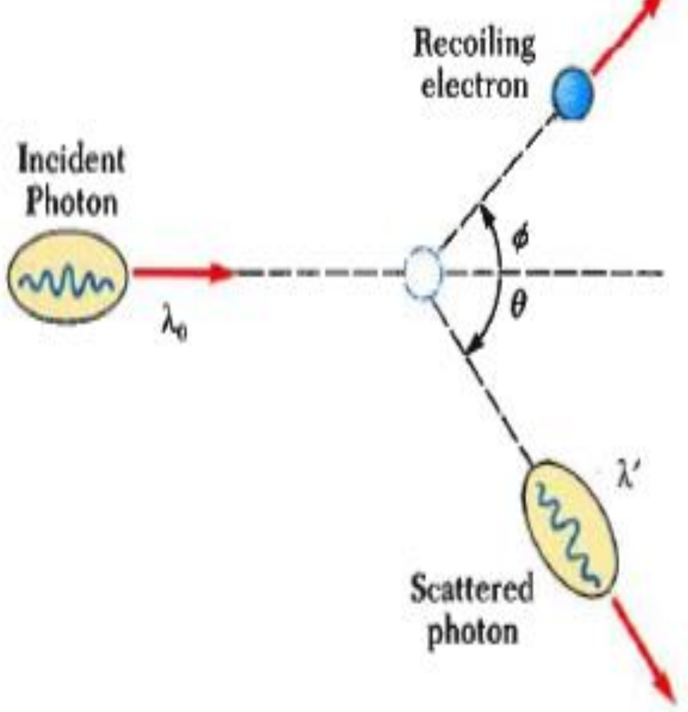


Figure 40.11 Diagram representing Compton scattering of a photon by an electron. The scattered photon has less energy (or longer wavelength) than the incident photon.

Energy of the incident photon

$$E_f = \hbar\omega_0$$

its impulse

$$\vec{p}_f = \hbar\vec{k}_0$$

$$\begin{aligned} \hbar\omega_0 + m_e c^2 &= \hbar\omega + \sqrt{p^2c^2 + m_e^2 c^4} \\ \hbar\mathbf{k}_0 &= \hbar\mathbf{k} + \mathbf{p}. \end{aligned}$$

$\lambda - \lambda_0 = \Lambda_C(1 - \cos\theta)$. $\Lambda_C = h/(mc) = 0{,}0242631$ Å.

We have seen that experiments such as blackbody radiation, the photoelectric effect, and Compton scattering can be explained using the photon picture of light, but not with the wave picture. However, it is important to realize that experiments such as diffraction and interference all need the wave picture, as a photon (particle) picture fails in these cases. Both pictures are needed in different circumstances; one says that light exhibits a **wave-particle duality**: Light has a dual nature; in some cases it behaves as a wave, and in other cases it behaves as a photon. This wave-particle duality is the basis of the quantum theory of light, and has some profound physical and philosophical implications which are still being debated today.

de Broglie Waves

In 1924 a young physicist, de Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave-particle duality. He proposed that ordinary ``particles'' such as electrons, protons, or bowling balls could also exhibit wave characteristics in certain circumstances.

The de Broglie relations

The de Broglie equations relate the wavelength λ and frequency f to the momentum p and energy E, respectively, as

$$\lambda = \frac{h}{p} \text{ and } f = \frac{E}{h}$$

where h is Planck's constant. The two equations are also written as

$$p = \hbar k$$

$$E = \hbar \omega$$

wavelength of a particle with mass *m* moving at speed *v*

$$\boxed{\lambda = \frac{h}{mv}}$$

because the momentum of such a particle is $p = mv$

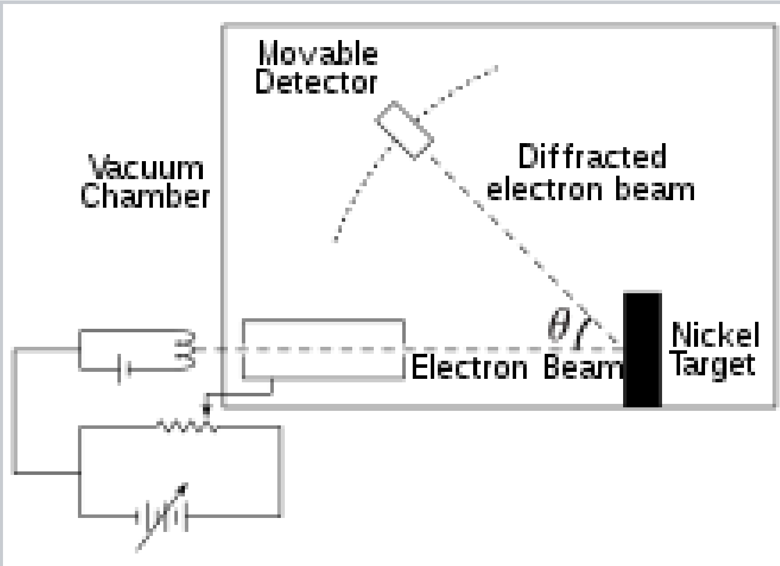
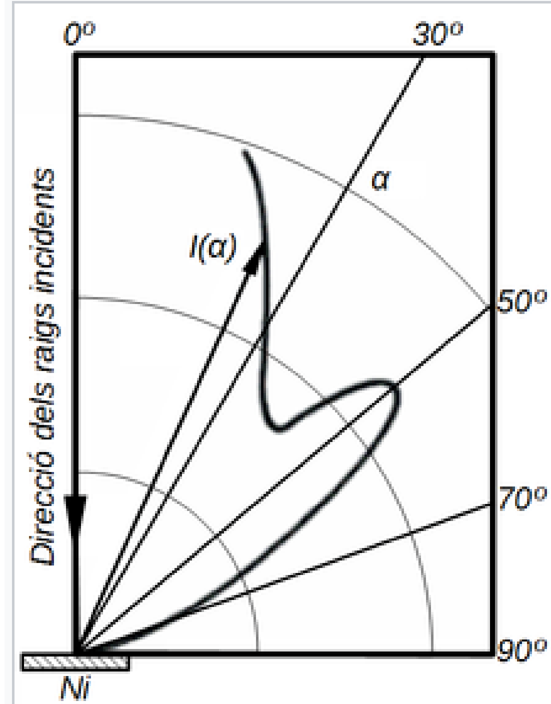


Схема аппарата, использованного Дэвиссоном и Джермером в 1927 году.



Интенсивность дифрагированных электронов при напряжении 54 В и запущенных перпендикулярно кристаллографической плоскости (111) относительно угла дифракции (полярные координаты).

Дифракция электронов, как и рентгеновские лучи, происходит в определённых предпочтительных направлениях, предполагающих участие нескольких слоёв параллельных плоскостей атомов никеля внутри кристалла. Из-за его малой длины рентгеновские лучи обладают хорошей проникающей способностью. Формула Брэгга имеет вид

$$2\,d\,\sin\theta = n\,\lambda$$

- d — расстояние между двумя кристаллографическими плоскостями;
- θ — угол дифракции, угол между падающим лучом и кристаллографическим направлением или плоскостью кристалла, участвующего в дифракции;
- n — порядок дифракции (1, 2, 3,. . .);
- λ — длина волны электронов[25].

Первоначально покоящаяся **частица** с зарядом e и массой m в результате прохождения разности потенциалов U **приобретает скорость** ϑ, которую **определяют из закона сохранения энергии**: $\frac{1}{2}m\vartheta^2 = eU$

Кинетическая энергия электронов,

ускоренных разностью потенциалов U равна eU.

Откуда $\vartheta = \sqrt{\frac{2eU}{m}}$. Длина волны де Бройля

$$\lambda = \frac{2\pi\hbar}{\sqrt{2emU}}$$

Для электрона $e = 1,6 \cdot 10^{-19}$ Кл, $m = 9,1 \cdot 10^{-31}$ кг.

$$\lambda = \sqrt{\frac{150}{U}} \cdot 10^{-10}\,\text{м} = 1,2/\sqrt{U}\ \text{нм.}$$

следует, что при энергиях электронов порядка нескольких эВ λ **де Бройля** имеет порядок 1нм, то есть порядок атомных расстоянии в кристаллах. Поэтому волновые свойства электронов при таких энергиях можно обнаружить в опытах по дифракции на кристаллах.

Experimental confirmation [edit]

Elementary particles [edit]

In 1928 at Bell Labs, Clinton Davisson and Lester Germer fired slow-moving electrons at a crystalline nickel target. The angular dependence of the reflected electron intensity was measured, and was determined to have the same diffraction pattern as those predicted by Bragg for x-rays. Before the acceptance of the de Broglie hypothesis, diffraction was a property that was thought to be only exhibited by waves. Therefore, the presence of any diffraction effects by matter demonstrated the wave-like nature of matter. When the de Broglie wavelength was inserted into the Bragg condition, the observed diffraction pattern was predicted, thereby experimentally confirming the de Broglie hypothesis for electrons.

This was a pivotal result in the development of quantum mechanics. Just as Arthur Compton demonstrated the particle nature of light, the Davisson-Germer experiment showed the wave-nature of matter, and completed the theory of wave-particle duality. For physicists this idea was important because it means that not only can any particle exhibit wave characteristics, but that one can use wave equations to describe phenomena in matter if one uses the de Broglie wavelength.

Since the original Davisson-Germer experiment for electrons, the de Broglie hypothesis has been confirmed for other elementary particles.

$$\psi = \psi_0 e^{i\left(\vec{k}\vec{r} - \omega t\right)} = \psi_0 e^{i\left(\frac{\vec{p}\vec{r}}{\hbar} - \frac{Et}{\hbar}\right)}$$

de Broglie wave

According Born square of absolute value of psi-function denotes the probability dp that the particle will be found in element dV:

$$dp = |\psi|^2 \, dV$$

$$\int dP = \int |\psi|^2 \, dV = 1$$

Normalization condition

$$|\psi(x, y, z)|^2$$

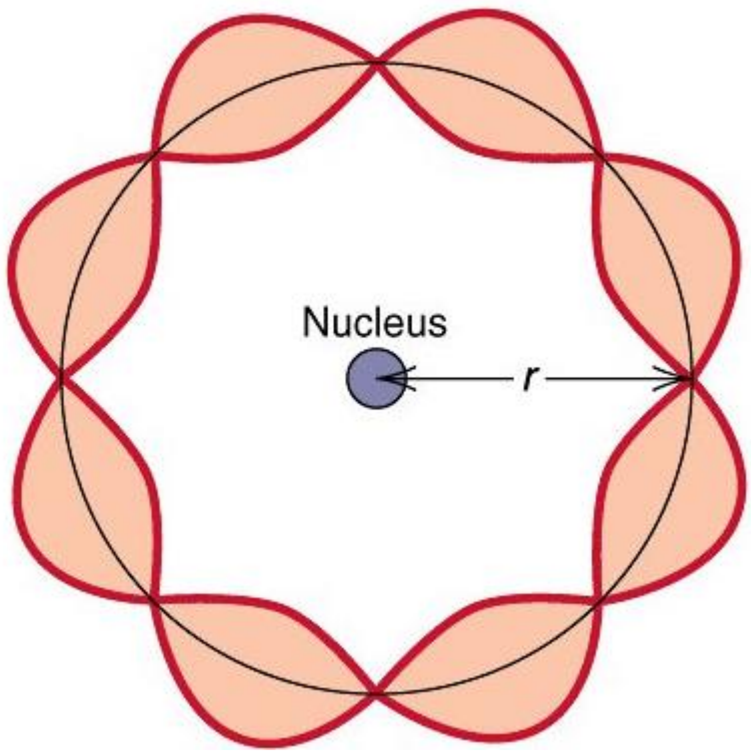
- Probability density of particle to be found in point (x, y, z)

$$|\psi(x, y, z)|^2 \, dxdydz$$

Probability of particle to be found in volume element $(dx \cdot dy \cdot dz)$

Conclusion

From sense of Psi- functions follows that the quantum mechanics has a statistical property. She doesn't allow to define a particle displacement in space. So, with reference to a microparticle concept of a trajectory loses sense. With the help of psi- functions can be predicted only, with what probability the particle can be found out in various points of space. The quantum mechanics opens true behavior of microparticles much more deeply.


Nucleus
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